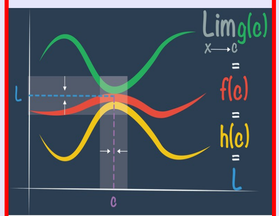
Calculus I

Lecture 49



Feb 19-8:47 AM

If $f(x) \geq 0$ and it is Continuous on $[a,b]$, then the area below $f(x)$ and above x-axis on $[a,b]$ is given by

$$A = \int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $f(x) = F'(x)$

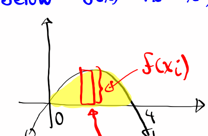
ex: Find the area below $f(x) = 4x - x^2$, and above x-axis

$f(x) = 4x - x^2$

$= x(4-x)$

Two x-Ints $(0,0), (4,0)$

Parabola $\cap$



$$A = \int_0^4 f(x)\,dx = \int_0^4 (4x - x^2)\,dx$$

$$= \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]\Big|_0^4$$

$$= \left(2x^2 - \frac{x^3}{3}\right)\Big|_0^4$$

$$= 2(4)^2 - \frac{4^3}{3} - 0$$

$$= 32 - \frac{64}{3} = \frac{96-64}{3} = \boxed{\frac{32}{3}}$$

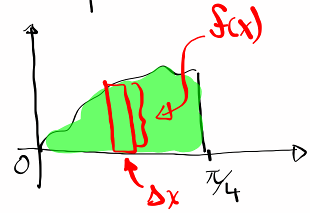
May 14-8:45 AM

Find the area below $f(x) = \sec x \tan x$, above x-axis from $x=0$ to $x=\frac{\pi}{4}$.

$0 \le x \le \pi/4$

we are in QI.

$\sec x \tan x \ge 0$ in QI



$$A = \int_0^{\pi/4} \sec x \tan x \, dx = \sec x \Big|_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0$$

$$= \boxed{\sqrt{2} - 1}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \rightarrow \sec\frac{\pi}{4} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\cos 0 = 1 \rightarrow \sec 0 = \frac{1}{1} = 1$$

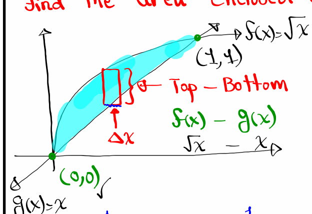
May 14-8:54 AM

Area between two Curves:

Suppose $f(x) \ge g(x)$ for all values on $[a,b]$ and both $f(x)$ and $g(x)$ are cont. on $[a,b]$, the area below $f(x)$ but above $g(x)$ is

$$A = \int_a^b [f(x) - g(x)] \, dx$$

Find the area enclosed by $f(x) = \sqrt{x}$ and $g(x) = x$.



$x = \sqrt{x}$

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x=0, \ x=1$

$$A = \int_0^1 (\sqrt{x} - x) \, dx = \int_0^1 (x^{1/2} - x) \, dx$$

$$= \left(\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right)\Bigg|_{x=0}^{x=1}$$

$$= \left(\frac{2}{3} x\sqrt{x} - \frac{1}{2}x^2\right)\Bigg|_0^1 = \frac{2}{3} \cdot 1 \cdot \sqrt{1} - \frac{1}{2}(1)^2 = \frac{2}{3} - \frac{1}{2}$$

$$= \frac{4-3}{6} = \boxed{\frac{1}{6}}$$

May 14-9:01 AM

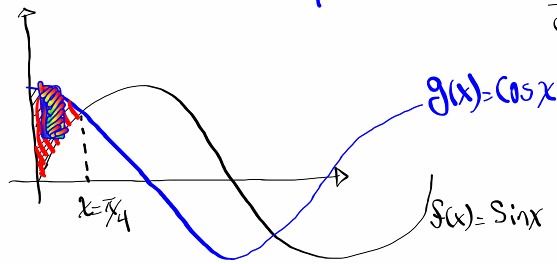
Find the area between $f(x) = \sin x$ and $g(x) = \cos x$ from $x = 0$ to $x = \frac{\pi}{4}$.

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}$



$$A = \int_0^{\pi/4} (\text{Top} - \text{Bottom})\,dx = \int_0^{\pi/4} (\cos x - \sin x)\,dx$$

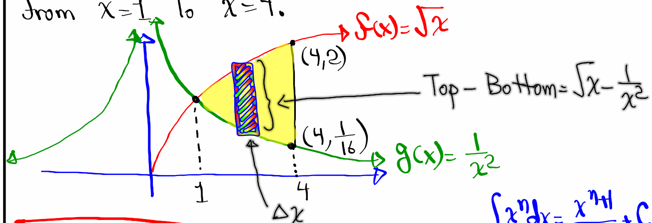
$$= \left[\sin x + \cos x\right]\Big|_{x=0}^{x=\pi/4} = \left[\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right] - \left[\sin 0 + \cos 0\right]$$

$\sin\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$, $\cos\frac{\pi}{4} \to \frac{\sqrt{2}}{2}$, $\sin 0 \to 0$, $\cos 0 \to 1$

$$= \boxed{\sqrt{2} - 1}$$

May 14-9:10 AM

Find the area bounded by $f(x) = \sqrt{x}$, $g(x) = \frac{1}{x^2}$ from $x = 1$ to $x = 4$.



$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$\boxed{A = \int_1^4 \left[\sqrt{x} - \frac{1}{x^2}\right] dx} = \int_1^4 \left(x^{1/2} - x^{-2}\right) dx$$

$$= \left(\frac{x^{3/2}}{3/2} - \frac{x^{-1}}{-1}\right)\Big|_1^4 = \left[\frac{2}{3}x\sqrt{x} + \frac{1}{x}\right]\Big|_1^4$$

$$= \left(\frac{2}{3}\cdot 4\sqrt{4} + \frac{1}{4}\right) - \left(\frac{2}{3}\cdot 1\sqrt{1} + \frac{1}{1}\right)$$

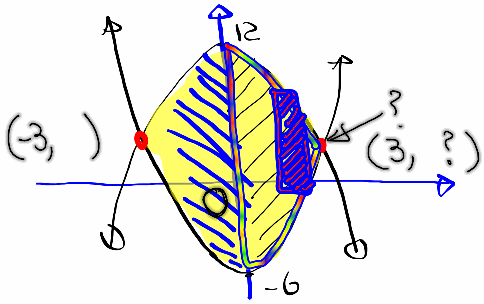
$$= \frac{16}{3} + \frac{1}{4} - \frac{2}{3} - 1 = \frac{14}{3} - \frac{3}{4} = \frac{56 - 9}{12}$$

$$= \boxed{\frac{47}{12}}$$

May 14-9:18 AM

Find the area of the region enclosed by $f(x) = 12 - x^2$ and $g(x) = x^2 - 6$.

$f(x)$ & $g(x)$ are both even functions.



$g(x) = f(x)$

$x^2 - 6 = 12 - x^2$

$x^2 + x^2 = 12 + 6$

$2x^2 = 18$

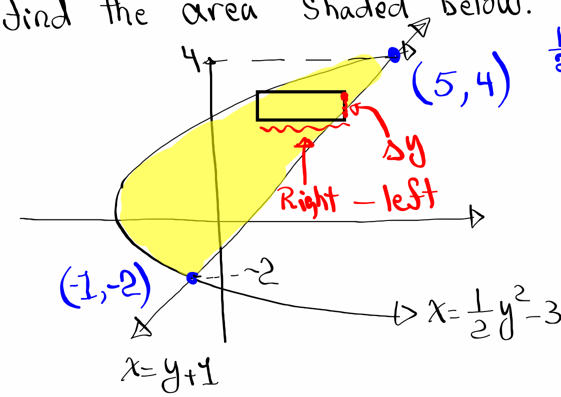
$x^2 = 9 \rightarrow x = \pm 3$

$$A = 2\int_0^3 [\underbrace{(12 - x^2)}_{\text{Top}} - \underbrace{(x^2 - 6)}_{\text{Bottom}}]dx$$

Make Sure to finish this.

May 14-9:29 AM

Find the area shaded below.



$\frac{1}{2}y^2 - 3 = y + 1$

$y^2 - 6 = 2y + 2$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4 \quad y = -2$

$$A = \int_{-2}^{4} [\text{Right} - \text{left}]dy = \int_{-2}^{4} \left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right]dy$$

$$= \int_{-2}^{4} \left[y - \tfrac{1}{2}y^2 + 4\right]dy = \left[\frac{y^2}{2} - \frac{1}{2}\cdot\frac{y^3}{3} + 4y\right]\Bigg|_{-2}^{4}$$

Make Sure to finish this.

May 14-9:37 AM

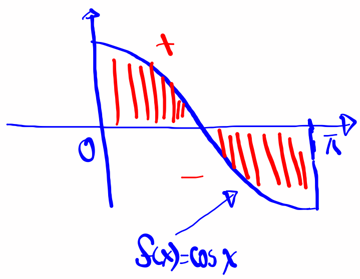
The average value for a cont. function $f(x)$ on $[a,b]$ is given by

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Find f_{ave} on $[0,\pi]$ for $f(x) = \cos x$.

$$f_{ave} = \frac{1}{\pi - 0}\int_0^\pi \cos x\,dx = \frac{1}{\pi}\Big[\sin x\Big]\Big|_0^\pi$$

$$= \frac{1}{\pi}\Big[\cancel{\sin\pi}^{\,0} - \cancel{\sin 0}^{\,0}\Big] = \boxed{0}$$



Average value